ANALYTICAL STUDY OF THE CHARACTERISTICS OF HYDRAULIC JUMP IN SLOPED TRIANGULAR CHANNELS

Ehab Fattouh[1] *and Ghada Ezizah*[2]

[1]*Associate Professor, Faculty of Engineering, Ain Shams University, Cairo, Egypt,*
E-mail: ihab_mostafa@ eng.asu.edu.eg
[2]*Associate Professor, Faculty of Engineering, Ain Shams University, Cairo, Egypt,*
E-mail: ghada_ezizah@ yahoo.com

ABSTRACT

Hydraulic jump is a phenomenon caused by the change in stream regime from supercritical to subcritical flow with considerable energy dissipation and local rise in depth of flow. Most of hydraulic jump researches focused on classical channels. In this paper, the characteristics of hydraulic jump in a sloped triangular channel of 90^{o} central angle has been studied analytically. Equations and design charts were provided to aid the designers in obtaining sequent depths directly when the loss of energy is known and vice versa. The proposed equations are of high accuracy and applicable to a wide range of Froude numbers and slopes. An experimental analysis was also proposed to find better formulation of the obtained equations. Moreover, a sensitivity analysis was performed to investigate the effect of the different parameters on the hydraulic jump characteristics.

Keywords: Sloping channels – Triangular sections – Hydraulic jump.

1 INTRODUCTION

The hydraulic jump is a beneficial mean of dissipating excess energy and change the type of flow to subcritical flow downstream a channel. If it takes place in a rectangular channel with a smooth horizontal bed, it is called classical hydraulic jump, which has been studied extensively (Abdel-Mageed, 2015). These studies include (Hager & Bretz, 1987), (Hager, 1992), and (Ead & Rajaratnam, 2002). On the other hand, the hydraulic jump in triangular channels has not received much attention. However, the rare studies in this field showed that concerning the energy dissipation and the sequent depths ratio, triangular channels are much more effective than rectangular channels, when the value of Froude number is fixed. (Hager & Wanoschek, 1985).

The triangular section fails to satisfy the requirement of a stilling basin, but practically, it can be used as an irrigation ditch (Achour, 1989). The function of ditches is to convey storm-water runoff from, through, or around roadway rights-of-way without damage. (Debabeche, et al., 2009) showed that the downstream water surface could be raised through the hydraulic jump in a triangular ditch. (Mays , 1999) concluded that the analysis and design principles of open channel flow could be applied for ditches.

Several researches were devoted to give an analytical solution of the specific energy and specific force equations for different cross-sections. (Vatankhah & Omid, 2010) gave direct solution of hydraulic jump to obtain sequent depths ratio in horizontal triangular channels. Also, (Rashwan, 2013) developed equations for hydraulic jump elements to obtain the sequent depths when the discharge and the energy loss are known in horizontal triangular channels.

On the other hand, reasonable numbers of studies were aimed to investigate the effect of bed slope on the characteristics of hydraulic jump in rectangular channels. These include (Abdel-Azim, 2003), (Beirami, 2006), (Abd-Elmegeed, 2015) and (Ghose D.K., et. al., 2019). They found that the slope has a great effect on the sequent depth, the relative hydraulic jump length, and the relative energy loss.

It should be mentioned that, few numbers of researchers studied the hydraulic jump through triangular channels, and limited numbers of them studied it through sloped bed. (Debabeche, et al., 2009) studied the hydraulic jump in a sloped triangular channel with a 90^0 central angle. They got a general relation for the sequent depths ratio as a function of the inflow Froude number and the channel slope. This relation is predicted a direct determination of the sequent depths ratio when Froude number and the channel slope are known. (Eltoukhy & Elkashef, 2016) also studied the hydraulic jump in a sloped bed through a triangular channel with a central angle of 90^o experimentally and developed empirical equations to estimate the characteristics of the hydraulic jump for given initial Froude number and bed slope.

The present study investigates theoretically and experimentally the hydraulic jump problem in sloped triangular channels with a central angle of 90^o. The configuration of the jump adopted for this study corresponds to the D-jump according to the classification of (Kindsvater, 1944). The study proposed a method to obtain the sequent depths when the discharge and the energy loss are known. Alternatively, the proposed method enables to estimate the energy loss when the discharge and one of the sequent depths are given. The objective of the experimental work is to find an expression for the length of the jump as a function of jump height in order to simplify the mathematical calculations of relative sequent depths (y1/yc & y2/yc) and dimensionless energy loss (EL/yc). The experimentally collected data will be used to verify the developed methods.

2 EXPERIMENTAL WORK

The experiments of this study were made in the hydraulic laboratory of the irrigation and hydraulics department at faculty of Engineering, Ain Shams University. These experiments were conducted in a flume 250 cm long, its cross section is symmetrical triangular with an apex angle 90^o and 15.0 cm wide of top width. The experimental work was carried out for six different slopes 0, 0.03, 0.06, 0.09, 0.12 and 0.15. For each slope six discharges values 1.0, 1.5, 2.0, 2.5, 3.0, and 3.5 lit/sec were performed.

There is a vertical sharp edge gate at the flume upstream and the water depth can be controlled with the help of a hand driven gear system. The hydraulic jump can be adjusted to be formed inside the flume through controlling the upstream gate and a tilting gate at the flume downstream. The water depth can be measured by means of point gauges mounted on instrument carriers with 1 mm accuracy. The sequent depths and the length of the hydraulic jump were measured for all values of discharge, bed slopes, and different gate openings.

On the basis of linear fitting between the measured length and the difference between the measured values of sequent depths (y_2-y_1), the experimental data were analyzed to obtain an empirical expression for the hydraulic jump length:

$$L = m\,(y_2 - y_1); \qquad m = 1.557\, S^{-0.575} \qquad (1)$$

Where m is a constant depended on the slope of the channel S and it was determined by regression using the experimental data.

3 THEORETICAL ANALYSIS

Design of stilling basins requires estimation of the characteristics of the hydraulic jump such as; sequent depths and the energy loss. For this purpose, the momentum and energy equations should be solved. This needs lengthy and complicated iterations.

3.1 Sequent Depths

The momentum equation is applied between section 1 and section 2 in a sloped triangular channel with a central angle of 90^o, as shown in Fig. 1. It is considered that the friction force is neglected and the pressure is hydrostatic.

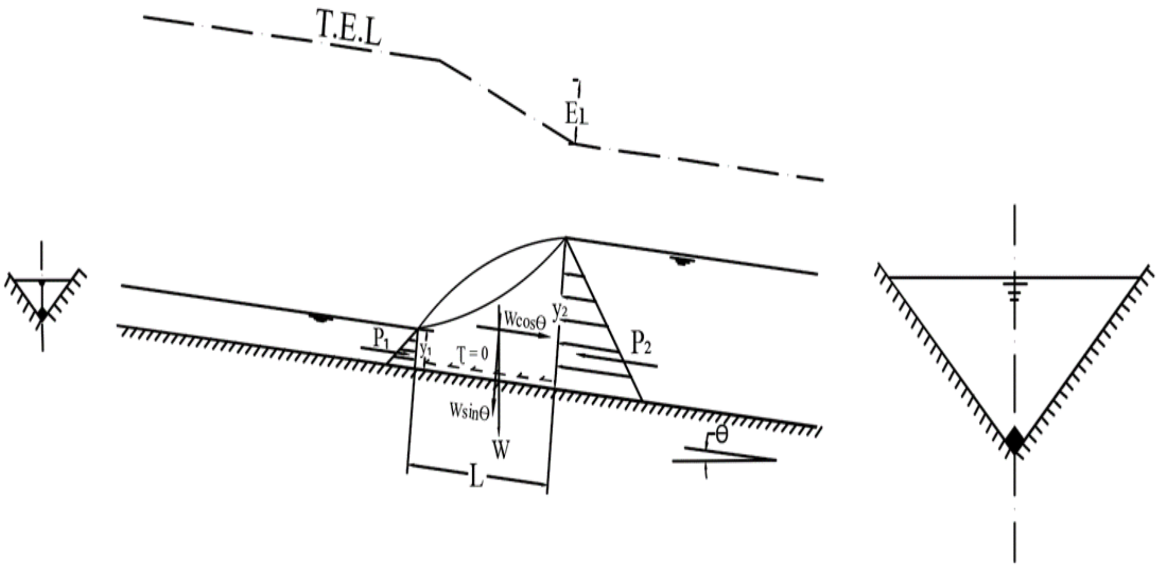

Figure 1. Definition sketch for a hydraulic jump in sloping triangular channel

$$P_1 + M_1 + F_W = P_2 + M_2 \qquad (2)$$

Where P_1, P_2, M_1 and M_2 are the hydrostatic and the momentum forces at sections 1 and 2 respectively; F_W is the component of the water weight between sections 1 and 2 in the flow direction.

$$\gamma A_1 \bar{h}_1 + \frac{\gamma}{g} Q \frac{Q}{A_1} + W \sin\theta = \gamma A_2 \bar{h}_2 + \frac{\gamma}{g} Q \frac{Q}{A_2} \qquad (3)$$

Where γ is the specific weight of the water; A_1 and A_2 are the water areas at sections 1 and 2, respectively; $\bar{h}_1$ and $\bar{h}_2$ are the vertical distances from the water surface to the centroids of sections 1 and 2, respectively; g is the gravity of acceleration; Θ is the angle of inclination of channel bed with regards to the horizontal; Q is the discharge through the channel.

$$A_1 \bar{h}_1 + \frac{Q^2}{g A_1} + V \sin\theta = A_2 \bar{h}_2 + \frac{Q^2}{g A_2} \qquad (4)$$

Where V is the water volume between sections 1 and 2. For a triangular section with side slope z; $A_1 = z y_1^2$; $A_2 = z y_2^2$; where y_1 and y_2 are the water depths at sections 1 and 2, respectively.

$$z y_1^2 \frac{y_1 \cos\theta}{3} + \frac{Q^2}{g z y_1^2} + V \sin\theta = z y_2^2 \frac{y_2 \cos\theta}{3} + \frac{Q^2}{g z y_2^2} \qquad (5)$$

Multiplying Eq. (5) by 2/z leads to:

$$2 \frac{y_1^3 \cos\theta}{3} + \frac{2 Q^2}{g z^2 y_1^2} + \frac{2V}{z} \sin\theta = 2 \frac{y_2^3 \cos\theta}{3} + \frac{2 Q^2}{g z^2 y_2^2} \qquad (6)$$

At the critical state: $\frac{Q^2}{g} = \frac{A_c^3}{T_c}$ so; $\frac{Q^2}{g} = \frac{(z\, y_c^2)^3}{2zy_c}$ and $y_c = \left(\frac{2\,Q^2}{g\,z^2}\right)^{1/5} \Rightarrow y_c^5 = \frac{2\,Q^2}{g\,z^2}$ (7)

Where A_c is the water area at the critical section; T_c is the top width at the critical section; and y_c is the critical water depth.

$$F_n^2 = \frac{v^2}{g\,D} = \frac{Q^2}{g\,A^2\frac{y}{2}} = \frac{Q^2}{g\,z^2 y^4 \frac{y}{2}} = \frac{2\,Q^2}{g\,z^2 y^5} \quad , \ \text{Since } y_c^5 = \frac{2\,Q^2}{g\,z^2} \quad \Rightarrow \quad F_n^2 = \frac{y_c^5}{y^5} = \frac{1}{Y^5} \tag{8}$$

Where F_n is Froude number; D is the mean hydraulic depth; v is the mean velocity; Y is the relative water depth y/y_c.

Substituting Eq. (7) into Eq. (6) leads to:

$$y_c^5\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) + \frac{2V}{z}\sin\theta = \frac{2}{3}\left(y_2^3 - y_1^3\right)\cos\theta \tag{9}$$

Multiplying Eq. (9) by $\frac{3\ y_1^2\, y_2^2}{(y_2 - y_1)\cos\theta}$

$$2\,y_1^2\,y_2^2\,(y_2 + y_1)^2 - 2\,y_1^3\,y_2^3 - \frac{3}{\cos\theta}(y_2 + y_1)\,y_c^5 - \frac{6\,V\,y_1^2\,y_2^2\tan\theta}{z\,(y_2 - y_1)} = 0 \tag{10}$$

Dividing Eq. (10) by y_c^6

$$2\,B^2\,(Y_2 + Y_1)^2 - \frac{3}{\cos\theta}(Y_2 + Y_1) - 2\,B^3 - \frac{6\,V\,B^2\tan\theta}{z\,(y_2 - y_1)\,y_c^2} = 0 \tag{11}$$

Where Y_1 is a relative initial depth = y_1/y_c; Y_2 is a relative final depth = y_2/y_c; B is a dimensionless parameter = Y_1Y_2.

Solving Eq. (11) as a second-degree equation leads to:

$$Y_2 + Y_1 = \frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + \frac{48\,V\,B^4\tan\theta}{z\,(y_2 - y_1)\,y_c^2}}}{4\,B^2} \tag{12}$$

Multiplying eq. (12) by Y_1

$$Y_1^2 - \frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + \frac{48\ V\ B^4\tan\theta}{z\ (y_2 - y_1)\,y_c^2}}}{4\,B^2} * Y_1 + B = 0 \tag{13}$$

Moreover, Eq. (13) is solved as a second-degree equation

$$Y_1 = \frac{1}{2}\left[\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + \frac{48\,V\,B^4\,\tan\theta}{z\,(y_2 - y_1)\,y_c^2}}}{4\,B^2}\right] - \sqrt{\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + \frac{48\,V\,B^4\,\tan\theta}{z\,(y_2 - y_1)\,y_c^2}}}{4\,B^2}\right]^2 - 4B}\right] \quad (14)$$

$$Y_2 = \frac{1}{2}\left[\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + \frac{48\,V\,B^4\,\tan\theta}{z\,(y_2 - y_1)\,y_c^2}}}{4\,B^2}\right] + \sqrt{\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + \frac{48\,V\,B^4\,\tan\theta}{z\,(y_2 - y_1)\,y_c^2}}}{4\,B^2}\right]^2 - 4B}\right] \quad (15)$$

$$V = L * k * z * \frac{y_1^2 + y_2^2 + y_1 y_2}{3} \quad (16)$$

Where L is the inclined length of the hydraulic jump and it is presented as $L = m\,(y_2 - y_1)$; m is a constant dependent on the channel slope S and it was determined experimentally as shown in the previous section; the coefficient k is the ratio between the weight of the real volume of the jump to the weight of the computed one and its value is k = 1.12 (Debabeche, et al., 2009).

Substituting Eq. (16) in Eqs. (14) and (15) leads to:

$$Y_1 = \frac{1}{2}\left[\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + 16\,m\,k\,J}}{4\,B^2}\right] - \sqrt{\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + 16\,m\,k\,J}}{4\,B^2}\right]^2 - 4\,B}\right] \quad (17)$$

$$Y_2 = \frac{1}{2}\left[\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + 16\,m\,k\,J}}{4\,B^2}\right] + \sqrt{\left[\frac{\frac{3}{\cos\theta} + \sqrt{\left(\frac{3}{\cos\theta}\right)^2 + 16\,B^5 + 16\,m\,k\,J}}{4\,B^2}\right]^2 - 4\,B}\right] \quad (18)$$

Where $J = \left(Y_1^2 + \frac{B^2}{Y_1^2} + B\right) B^4 \sin\theta\ \sec^2\theta = (Y_2^2 + \frac{B^2}{Y_2^2} + B)\, B^4 \sin\theta\ \sec^2\theta$

Figure 2 is a plot of Eq. (17) which presents the relation between the relative initial depth Y_1 and the dimensionless parameter B, while Fig. 3 is a plot of Eq. (18) and presents the relation between the relative final depth Y_2 and B.

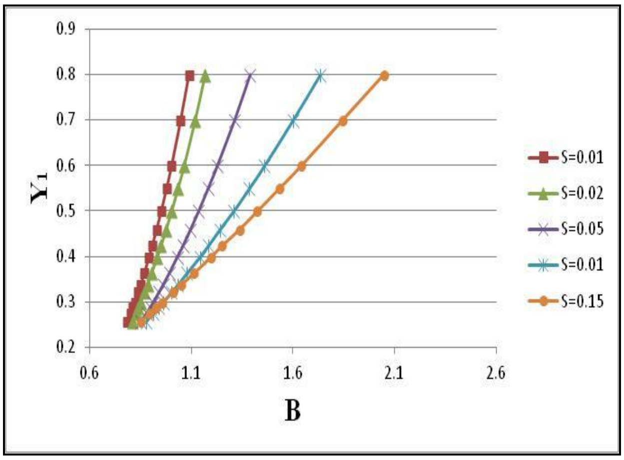

Figure 2. Relationship between Y1 and B, Eq. (17)

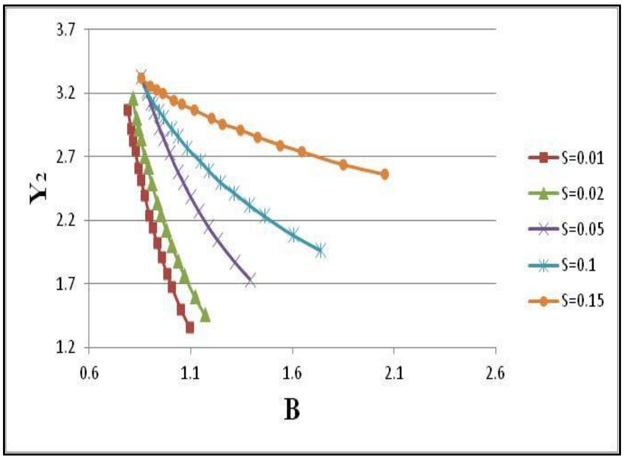

Figure 3. Relationship between Y2 and B, Eq. (18)

Eqs. (19), (20), (21), and (22) were developed based on the data of Fig. 2 and Fig. 3 using least square technique. Eqs. (19) and (21) are approximate relation between the relative initial depth Y_1 and dimensionless parameter B for bed slope $S \leq 0.1$ and $S > 1$, respectively. Also, Eqs. (20) and (22) are approximate relation between the relative final depth Y_2 and B for $S \leq 0.1$ and $S > 1$, respectively. R^2 for all of them exceeds 0.97.

$$Y_1 = \left(0.196\ S^{-0.22}\right) B\ \mathrm{EXP}\left(-3.2\ S^2 + 1.319\ S + 0.19\right) \quad \text{for } (S \leq 0.1) \tag{19}$$

$$Y_2 = \left(-141.3\ S^2 + 32.73\ S + 1.39\right) B\ \mathrm{EXP}\left(-3.7\ S^2 - 0.777\ S + 3.36\right) \quad \text{for } (S \leq 0.1) \tag{20}$$

$$Y_1 = \left(130.5\ S^2 - 36.33\ S + 3.858\right) B\ \mathrm{EXP}\left(2.97\ S^2 - 5.91\ S + 2.11\right) \quad \text{for } (S > 0.1) \tag{21}$$

$$Y_2 = \left(-130.7\ S^2 + 36.36\ S - 2.855\right) B\ \mathrm{EXP}\left(-12.2\ S^2 + 8.91\ S - 1.35\right) \quad \text{for } (S > 0.1) \tag{22}$$

3.2 Energy Loss

The energy losses by the jump can be calculated as $E_L = E_1 - E_2$, where E_1 and E_2 is the total energy at the sections before and after the jump respectively

$$E_L = \left(E_1 + L \tan\theta\right) - E_2 = \left(y_1 \cos\theta + \frac{Q^2}{2 g A_1^2}\right) + L \tan\theta - \left(y_2 \cos\theta + \frac{Q^2}{2 g A_2^2}\right) \tag{23}$$

Substituting from Eq. (6) into Eq. (23)

$$E_L = \frac{\cos\theta}{6\, y_1^2\, y_2^2}\left[6\, y_1^3 y_2^2 - 6\, y_1^2 y_2^3 + y_2^5 + y_2^3\, y_1^2 - y_1^3\, y_2^2 - y_1^5 - \sin\theta\, \sec^2\theta * L\, k\, (y_1^2 + y_2^2 + y_1 y_2)\,(y_2^2 + y_1^2) + L \sin\theta\, \sec^2\theta * 6\, y_1^2 y_2^2\right] \tag{24}$$

Dividing equation (24) by y_c

$$\frac{E_L}{y_c} = \frac{\cos\theta}{6\, Y_1^2 Y_2^2}\left[5 Y_1^3 Y_2^2 - 5 Y_1^2 Y_2^3 + Y_2^5 - Y_1^5 - \sin\theta\, \sec^2\theta * \frac{L}{y_c} k\, (Y_1^2 + Y_2^2 + Y_1 Y_2)(Y_2^2 + Y_1^2) + \frac{L}{y_c} \sin\theta\, \sec^2\theta * 6\, Y_1^2 Y_2^2\right] \tag{25}$$

Substituting Y_2 by B/Y_1 to have a relation between E_L and Y_1

$$\frac{E_L}{y_c} = \frac{\cos\theta}{6\,B^2}\left[5\,B^2Y_1 - 5\,\frac{B^3}{Y_1} + \frac{B^5}{Y_1^5} - Y_1^5 - \sin\theta\,\sec^2\theta * m\,k\left(\frac{B}{Y_1} - Y_1\right)\left(Y_1^2 + \frac{B^2}{Y_1^2} + B\right)\left(\frac{B^2}{Y_1^2} + Y_1^2\right) + m\left(\frac{B}{Y_1} - Y_1\right)\sin\theta\,\sec^2\theta * 6\,B^2\right] \tag{26}$$

Substituting Y_1 by B/Y_2 to have a relation between E_L and Y_2

$$\frac{E_L}{y_c} = \frac{\cos\theta}{6\,B^2}\left[5\,\frac{B^3}{Y_2} - 5\,B^2Y_2 + Y_2^5 - \frac{B^5}{Y_2^5} - \sin\theta\,\sec^2\theta * m\,k\left(\frac{B}{Y_2} - Y_2\right)\left(\frac{B^2}{Y_2^2} + Y_2^2 + B\right)\left(\frac{B^2}{Y_2^2} + Y_2^2\right) + m\left(\frac{B}{Y_2} - Y_2\right)\sin\theta\,\sec^2\theta * 6\,B^2\right] \tag{27}$$

Eqs. (26) and (27) are plotted in Fig. 4 and Fig. 5, respectively and eq. (28) was developed based on the data of these figures using least square technique. The equation represents approximate relation between the dimensionless energy loss E_L/y_c and B. (R^2 exceeds 0.95).

$$B = (-1.25\,S - 0.035)\,\ln\frac{E_L}{y_c} + (-8.088\,S^2 + 6.292\,S + 0.934) \tag{28}$$

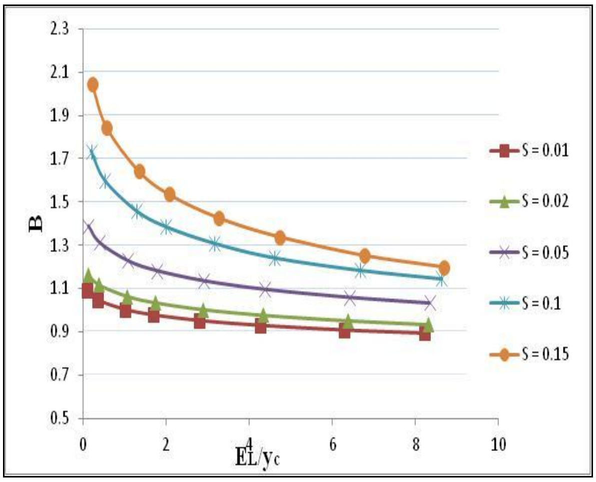

Figure 4. Relationship between B and E_L/y_c, Eq. (26), $E_L/y_c < 10$

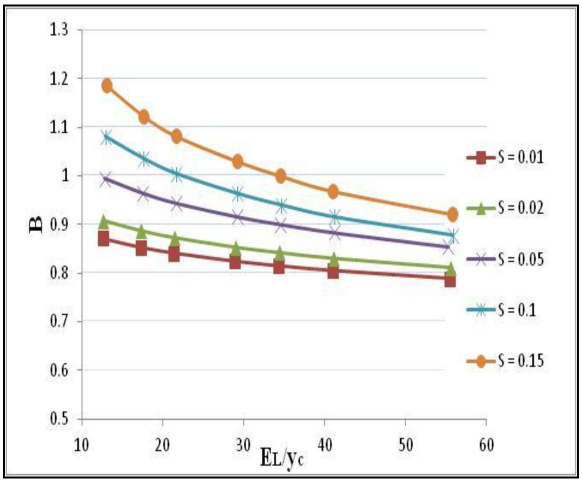

Figure 5. Relationship between B and E_L/y_c, Eq. (26), $E_L/y_c > 10$

4 VERIFICATION OF THE PROPOSED METHOD

Series of calculations were performed as shown in table 1 and the results were analyzed using least square technique to obtain direct relations between the sequent depths ratio y_2/y_1 and relative energy loss E_L/E_1 as a function of Froude number F_1 and channel slope S. It is essential to be able to compare with previous studies and permit direct determination of the sequent depth ratio y_2/y_1 and E_L/E_1. These relations can be expressed as following:

$$y_2/y_1 = (8.834\,S + 1.078)\,F_1^{(-1.367\,S + 0.699)} \tag{29}$$

For $F_1 \leq 10$, and $S \leq 0.1$

$$E_L/E_1 = (6.176\,S^2 - 1.223\,S + 0.442)\,\mathrm{Ln}\,F_1 + (1.339\,S^2 - 0.496\,S - 0.144) \tag{30}$$

For $F_1 \leq 10$ and $S > 0.1$

$$E_L/E_1 = (0.423\ S^2 - 0.115\ S + 0.0.389)\ Ln\ F_1 + (-1.263\ S^2 + 0.305\ S - 0.199) \quad (31)$$

For $F_1 > 10$ and $S \leq 0.\ 1$

$$E_L/E_1 = (0.361\ S + 0.146)\ Ln\ F_1 + (-\ 1.480\ S + 0.469) \quad (32)$$

For $F_1 > 10$ and $S > 0.\ 1$

$$E_L/E_1 = (1.263\ S^2 - 0.305\ S + 0.2)\ Ln\ F_1 + (-\ 2.754\ S^2 + 0.531\ S + 0.3) \quad (33)$$

Table. 1 Sample of calculations for S = 0.1

Y_1	B	Y_2	y_2/y_1	F_1	E_L/y_c	E_L/E_1
0.25	0.878	3.422	13.34	30	55.86	0.928
0.3	0.963	3.211	10.705	20.286	29.216	0.88
0.4	1.145	2.863	7.158	9.882	8.63	0.726
0.5	1.309	2.619	5.237	5.657	3.154	0.527
0.6	1.461	2.434	4.057	3.586	1.274	0.334
0.7	1.600	2.286	3.266	2.439	0.522	0.183
0.8	1.734	2.165	2.71	1.747	0.197	0.083

Fig. 6 and Fig.7 show sample of the comparisons of y_2/y_1 and E_L/E_1 estimated from the present study and from the equations developed by (Debabeche, et al., 2009) and (Eltoukhy & Elkashef, 2016). It is observed that, the present results are very close to the data of (Debabeche, et al., 2009) and also have a good agreement with the results from (Eltoukhy & Elkashef, 2016).

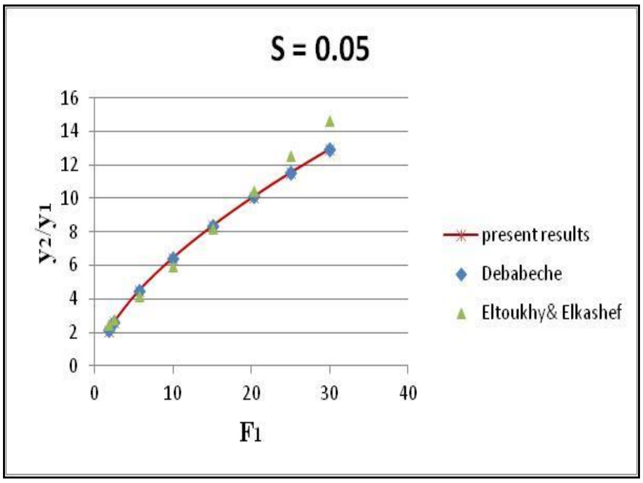

Figure 6. Relationship between y_2/y_1 and F_1 in comparison with other authors

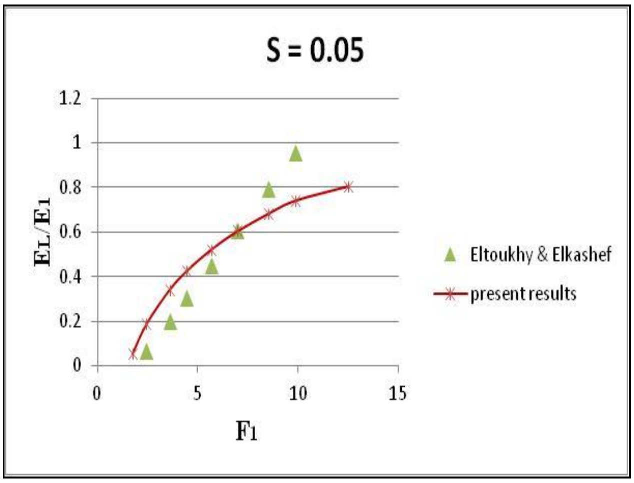

Figure 7. Relationship between E_L/E_1 and F_1 in Comparison with other authors

5 SENSITIVITY ANALYSIS

Sensitivity analysis is helpful to determine how sensitive the situation is to several factors of concern, so that the proper weight and consideration may be assigned to them. In this study, sensitivity analysis was carried out in a sloped triangular channel with $F_1 = 6.$ and $S = 0.1$ as a case study, to investigate the effect of change of Froude number F_1 and the channel bed slope S on the sequent depths ratio y_2/y_1 and the relative energy loss E_L/E_1.

From Fig. 8 and Fig. 9, it is noticed that the change in Froude number values has the biggest effect on both the sequent depths ratio and the relative energy loss. It means that the hydraulic jump characteristics y_2/y_1 and E_L/E_1 are much more sensitive to the change in the values of F_1 rather than the values of S.

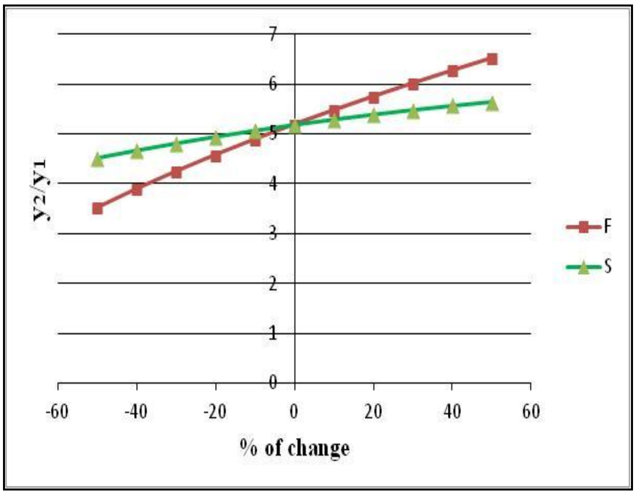

Figure 8. Sensitivity Analysis for y_2/y_1

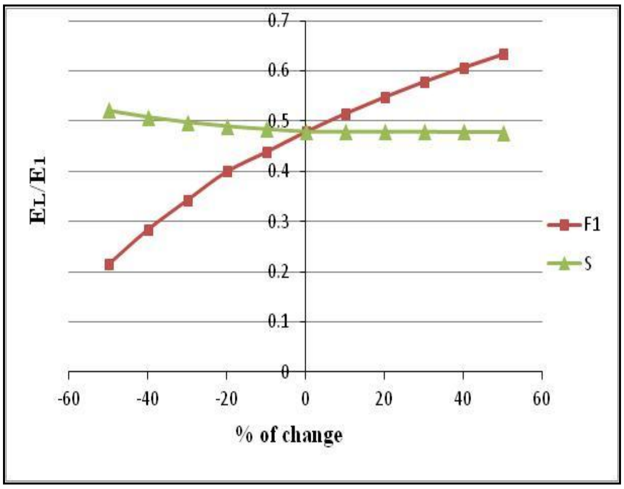

Figure 9. Sensitivity Analysis for E_L/E_1

6 PRACTICAL EXAMPLE

For design, the energy loss can be estimated as a percentage of the critical energy which presented as a dimensionless term (E_c/y_c) and it is equal to 1.25 in triangular channels. The following examples present the utility of the developed equations in two cases as describe below.

Case (1)

For a sloped triangular channel of 90^o central angle, the slope S = 0.1, the discharge Q = 3.5 m^3/s, and the estimated energy loss for the design = 2.5 (E_c/y_c). It is required to get the sequent depths.

Determine the critical depth from eq. (7), (y_c = 1.20 m); compute the dimensionless energy loss E_L/y_c = 2.5 x 1.25 = 3.125 ; by knowing E_L/y_c, B can be obtained from eq. (28) or fig. 4, (B = 1.3); by knowing B, the relative initial depth Y_1 can be obtained from the simplified eq. (19) or Fig. 2, (Y_1 = 0.486) with relative error = 2.83 % compared with the exact solution, and $y_1 = Y_1$ x y_c = 0.584 m; compute the relative final depth $Y_2 = B/Y_1$ = 2.675, and $y_2 = Y_2$ x y_c = 3.213 m.

Case (2)

For a sloped triangular channel of 90^o central angle, the slope S = 0.03, the discharge Q = 3.4 m^3/s, and the initial depth y_1 = 0.59 m. It is required to get the final depth and the relative energy loss.

Determine the critical depth from eq. (7), (y_c = 1.18 m); compute $Y_1= y_1/y_c$ = 0.50; Froude no F_1 can be obtained from eq. (8), (F_1 = 5.562); by knowing F_1 and S, the sequent depths ratio y_2/y_1 can be obtained from eq. (29), (y_2 = 2.477 m) with relative error = 1 % compared with the exact solution; by knowing F_1 and S, the relative energy loss E_L/E_1 can be obtained from eq. (30), (E_L/E_1 = 0.554) with relative error 2.3 % compared with the exact solution.

7 CONCLUSION

The hydraulic jump problem in a sloped triangular channel with 90^o central angle was studied. The study proposed a method to obtain the sequent depths y_1 and y_2 using dimensionless variable B, which is a function of dimensionless energy loss (E_L/y_c). When the discharge and E_L/y_c are known, the sequent depths can be accurately calculated using the proposed equations. These equations are of high accuracy and have a good agreement with previous studies.

The output results from the proposed method were analyzed to develop empirical relations between the hydraulic jump characteristics sequent depths ratio (y_2/y_1) and relative energy loss (E_L/E_1) from one side, and the inflow Froude number F_1 and the slope of the channel S from the other side. These relations permit direct determination of the sequent depth ratio y_2/y_1 and E_L/E_1, knowing F_1 and S. Also, a sensitivity analysis was carried out and it is concluded that the hydraulic jump characteristics y_2/y_1 and E_L/E_1 are much more sensitive to the change in the values of F_1 rather than the values of S.

REFERENCES

Abdel-Mageed N. B. (2015) Effect of Channel Slope on Hydraulic Jump Characteristics. *Physical Science International Journal*. PSIJ.2015. 7(4): 223-233.

Achour, B. (1989) Jump flowmeter in a channel of triangular cross-section without weir. *Journal of Hydraulic Research,* 27(2): 205– 214.

Beirami, M. and Chamani M. (2006) Hydraulic jumps in sloping channels: Sequent depth ratio. *Journal of Hydraulics Eng.*; 132(10):1061–1068.

Debabeche M., Cherhabil S., Hafnaoui A., and Achour A. (2009) Hydraulic jump in a sloped triangular channel. *Canadian Journal of Civil Engineering* 2009*,* 36:655–658.

Ead, S. A. and Rajaratnam, N. (2002) Hydraulic jump on corrugated beds. *Journal of Hydraulic Engineering,* 128(7): 656–663. doi:10.1061/(ASCE)0733-9429(2002)128:7(656).

Eltoukhy M. A. and Elkashef A. T. (2016) Field Experiments on Hydraulic Jump in a Sloped Triangular Channel. *British Journal of Applied Science BJAST& Technology* ; 18(2): 1-7.

Ghose D.K., Mandal P., and Samantaray S. (2019) Experimental Study of Hydraulic Jumps in an Inclined Rectangular Flume. *Pertanika Journal of Science & Technolgy 27 (1): 397 – 407.*

Hager, W. H. and Bretz, N. V. (1987) Hydraulic jumps at positive and negative steps. *Journal of Hydraulic Research,* 24(4): 237– 253. Discussion 25(3): 407–413.

Hager, W. H. (1992) Energy dissipators and hydraulic jump. *Kluwer Academic Publ., Water Science and Technology Library,* Dordrecht, Netherlands. Vol. 8, 288 p.

Hager W. H. and Wanoschek R. (1985) Hydraulic jump in a triangular channel. *Journal of Hydraulic Research.*; 25 (5):549-564.

Kindsvater, C.E. (1944) The hydraulic jump in sloping channels. *Transactions ASCE*, 109:1107-1154

Ljubiˇci R., Zindovi´c B., Vojt P., Pavlovi´c D., Kapor R., and Savi´c L. (2018) Hydraulic Jumps in Adverse-Slope Stilling Basins for Stepped Spillways. *Water*; 10, 460; doi:10.3390/w10040460

Mays Larry W. *Hydraulic design handbook.* McGraw Hill; 1999.

Negm, A. M. (2003) Analysis and formulation of hydraulic in sloping rectangular closed conduits. *Seventh International Water Technology Conference* Egypt.

Rashwan, I. M. H. (2013) Analytical solution to problems of hydraulic jump in horizontal triangular channels. *Ain Shams Engineering Journal*; 4: 365–368.

Vatankhah Ali R, Omid, M. H. (2010) Direct solution to problems of hydraulic jump in horizontal triangular channels. *Applied Math Lett*; 23: 1104–8.

LIST OF SYMBOLS

B	dimensionless parameter = Y_1Y_2
E_1	total energy at the section upstream the hydraulic jump [m]
E_2	total energy at the section downstream the hydraulic jump [m]
E_L	energy losses by the hydraulic jump [m]
E_L/E_1	relative energy loss
E_L/y_c	dimensionless energy loss
E_c/y_c	dimensionless critical energy
F_1	inflow Froude number
g	gravity of acceleration [$m.s^{-2}$]
L	inclined length of the hydraulic jump [m]
Q	discharge through the channel [$m^3.s^{-1}$]
S	channel bed slope
y_1	initial depth of the hydraulic jump [m]
y_2	final depth of the hydraulic jump [m]
y_c	critical water depth [m]
Y	relative water depth = y/y_c
Y_1	relative initial water depth = y_1/y_c
Y_2	relative final water depth = y_2/y_c
y_2/y_1	sequent depths ratio
z	cross section side slope
θ	angle of inclination of the channel bed with regard to the horizontal [rad]